Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Prediction of the Turbulent Near-Wake of a Symmetrical Aerofoil

P. BRADSHAW*
National Physical Laboratory, Teddington, England

Nomenclature

K_1 = "universal" constant in wake mixing-length formula for inner wake
L = dissipation length parameter, nominally $(\tau/\rho)^{3/2}$/(dissipation)
l = mixing length, defined as $(\tau/\rho)^{1/2}/(\partial U/\partial y)$
U = velocity in x direction
U_1 = U at edge of boundary layer
x = distance along wake centre line from trailing edge
y = distance normal to wake centre line
y_c = distance from centre line to the outgoing characteristic that starts at the trailing edge
δ_{995} = boundary layer thickness (to $U/U_1 = 0.995$)
δ_1 = displacement thickness
δ_2 = momentum thickness

Analysis

BOTH the model equations of Ref. 1 and theoretical work[2,3] on the effects of a change of surface roughness on a turbulent boundary layer indicate that (apart from a displacement effect) the influence of a disturbance at the surface is confined to a new "inner boundary layer" whose origin is at the point of disturbance. In this paper we treat the presence of a trailing edge as an extreme case of such a disturbance. In the model of Ref. 1, which leads to hyperbolic equations, the edge of the inner boundary layer is the outgoing characteristic that leaves the surface at the point of disturbance. If $y/\delta_{995} < 0.2$ the only independent length scale in an ordinary boundary layer is y; therefore if the thickness of the inner boundary layer, y_c, is less than $0.2\delta_{995}$ the disturbed flow can be described by a length scale y and a parameter y/y_c, leading to "self-preserving" (similar) forms for the perturbations of velocity and shear stress.[2] Strictly, this simple state of affairs exists only if the disturbance is small enough for "history" effects on the turbulence structure to be neglected but it should be a good first approximation in practice. Since dy_c/dx is usually about 0.03 if the disturbance is small,[1] y_c is less than $0.2\delta_{995}$ for $x < 7\delta_{995}$ approximately. Now within the inner boundary layer, dimensional analysis shows that any dependent length scale, L say, can be written as $L/y = f_1(y/y_c)$ and any dimensionless quantity can be written as $f_2(y/y_c)$. If we can use experimental data in one wake to determine the dependence on y/y_c of the length scale L and the dimensionless quantities a_1 and G used in the method of Ref. 1, we can then calculate any other (symmetrical) wake near enough to the trailing edge for y_c to be less than $0.2\delta_{995}$. In fact, we do not yet have enough data to do this: all we can do is to take advantage of the fact that in the inner layer the shear stress equation of Ref. 1 reduces to the "mixing length" formula, so that $L = (\tau/p)^{1/2}/(\partial U/\partial y)$ and use measurements of τ and U to deduce the best choice for $L/y = f_1(y/y_c)$. For $y > y_c$, $L = Ky \approx 0.4y$ as usual; the rest of the shear layer is calculated in the same way as an ordinary boundary layer, the only other difference being that the log-law boundary condition at $y = 0$ is replaced by $\tau = 0$, $\partial U/\partial y = 0$. The wake could of course be predicted by adaptations of other methods of boundary-layer calculation, and Nee and Kovasznay (private communication) have extended their method[5] to this problem: the advantage of the method of Ref. 1 is that it gives an explicit value for y_c. Calculations for the whole length of the wake would require more experimental data. According to Ref. 4 the pressure gradient normal to the wake is small, even at the trailing edge.

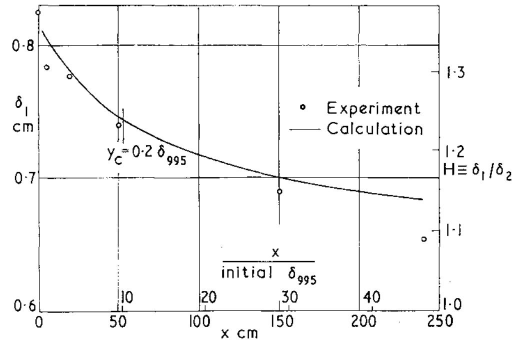

Fig. 1 Comparison with the data of Ref. 6: displacement thickness and shape parameter.

A "Mixing Length" Fit to the Data of Ref. 6

In any symmetrical wake, the velocity gradient and shear stress both vary as y near $y = 0$, so that the mixing length, $l \equiv (\tau/p)^{1/2}/(\partial U/\partial y)$, varies as $y^{-1/2}$ (in an asymmetric wake, l is imaginary in part of the flow). In the undisturbed part of the near-wake ($y > y_c$) we expect $L = 0.4y$ as in a boundary layer. According to the dimensional arguments in the last section, we may choose $L = 0.4yf(y/y_c)$, where $f(y/y_c) = K_1 (y/y_c)^{3/2}$ for small y/y_c and $f = 1$ for $y > y_c$. The only wake measurements that include shear stress profiles are those of Chevray and Kovasznay,[6] which enable us to find the constant K_1. The detailed behaviour of the length L in the region of transition between $L \propto y^{-1/2}$ and $L \propto y$ does not seem to be very critical: therefore we have assumed a sharp transi-

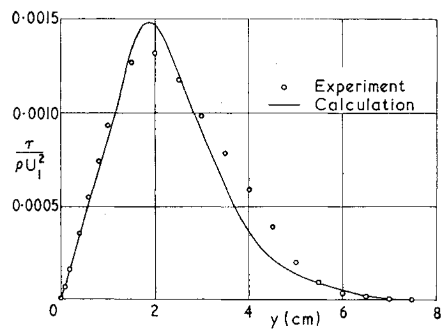

Fig. 2 Comparison with the data of Ref. 6: shear stress profile at x = 50 cm (9.6 initial δ_{995}).

Received August 28, 1969.

* Professor and Head of Mechanical Engineering Department; present address: Department of Aeronautics, Imperial College of Science and Technology, London, England.

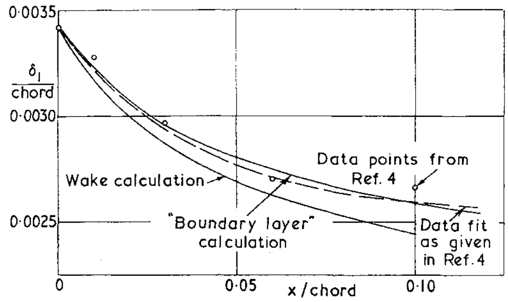

Fig. 3 Comparison with the data of Ref. 4: displacement thickness.

tion at the intersection of the two curves. We have chosen $K_1 = 0.26$ to give the best agreement between Chevray's data and calculations by the method of Ref. 1, using a length scale L equal to $0.4\ yK_1(y/y_c)^{-3/2}$ near $y = 0$—which is virtually the same as using a mixing length varying in the same way. The results for $K_1 = 0.26$ are shown in Figs. 1–2. The downstream limit of validity, the value of x at which y_c reaches 0.2δ, is marked on Fig. 1. Fig. 2 shows the shear stress profile at about this value of x. Chevray's measurements were at rather low Reynolds number, about 2000 based on momentum thickness, so that the optimum value of K_1 probably contains an inbuilt allowance for a thick viscous sublayer, though the effect of the latter on $d\delta_1/dx$ would be felt only for a few sublayer thicknesses downstream of the trailing edge.

Figure 3 shows a comparison, using $K_1 = 0.26$, with the data of Firmin[4] for an R.A.E. 101 aerofoil at zero incidence. At the trailing edge, δ_{995} is about 0.018 of the chord. The broken line in Fig. 3 is the data fit as given in Ref. 4, including points further downstream. The agreement is not as good as with Chevray's data, but Firmin did not measure shear stress profiles and the starting conditions for the wake calculation are rather uncertain (for details, see Ref. 7). The fact that the experiments were done at $M = 0.4$ and the calculations at $M = 0$ should not have affected the results appreciably. We have not yet written a program for the compressible wake but an indirect check on compressibility effects was made by doing a boundary-layer calculation, with the same pressure distribution and initial conditions as in the wake, at $M = 0.4$ and $M = 0$: the incompressible shape factor $H \equiv \delta_1/\delta_2$ (ignoring density changes), agreed to within about 0.005. This calculation revealed that the variation of displacement thickness in the "boundary layer" and wake calculations was almost identical (in fact the boundary layer values are closer to the experimental data). This is undoubtedly a coincidence; it is far from true for Chevray's constant-pressure wake, but the same may happen in other aerofoil wakes in strong favourable pressure gradient.

The simple "mixing length" fit used here is not valid once the inner wake has spread outside the inner layer of the boundary layer. In the outer layer, y_c/δ_{995} is an extra parameter, changes in the energy-diffusion function G will occur and, most important of all, the flow will no longer be self-preserving in Townsend's sense. A comparison[7] of the calculated shear stress profiles with Chevray's measurements shows that large errors accumulate for $x > 50$ cm. However, the mixing length fit should be valid far enough downstream for displacement surface calculations, and a more refined treatment must await more data on the turbulence structure of wakes. In asymmetrical wakes, separate calculations for each side will give a first approximation to δ_1. To predict the profiles we need data on the interaction between opposing shear layers.

References

[1] Bradshaw, P., Ferriss, D. H., and Atwell, N. P., "Calculation of Boundary Layer Development Using the Turbulent Energy Equation," *Journal of Fluid Mechanics*, Vol. 28, No. 3, May 1967, pp. 593–616.

[2] Townsend, A. A., "The Flow in a Turbulent Boundary Layer After a Change in Surface Roughness," *Journal of Fluid Mechanics*, Vol. 28, No. 2, Oct. 1966, pp. 255–266.

[3] Robinson, J. L., "Similarity Solutions in Several Turbulent Shear Flows," Aero Rept. 1242, Aug. 1967, National Physical Laboratory, Teddington, England.

[4] Firmin, M.C.P. and Cook, T. A., "Detailed Exploration of the Compressible Viscous Flow over Two-Dimensional Aerofoils at High Reynolds Numbers," Tech. Memo. Aero. 1076, July 1968, Royal Aircraft Establishment, Farnborough, England.

[5] Nee, V. W. and Kovasznay, L. S. G., "Simple Phenomenological Theory of Turbulent Shear Flows," *The Physics of Fluids*, Vol. 12, No. 3, March 1969, pp. 473–484.

[6] Chevray, R. and Kovasznay, L. S. G., "Turbulence Measurements in the Wake of a Thin Flat Plate," *AIAA Journal*, Vol. 7, No. 8, Aug. 1969, pp. 1641–1643.

[7] Bradshaw, P., "Calculation of Boundary Layer Development Using the Turbulent Energy Equation. V: Wakes Near a Trailing Edge," Aero Rept. 1285, Jan. 1969, National Physical Laboratory, Teddington, England.

Correlation between Turbulent Shear Stress and Turbulent Kinetic Energy

P. T. Harsha* and S. C. Lee†
ARO Inc., Arnold Air Force Station, Tenn.

Nomenclature

a_1 = constant of proportionality in relation between turbulent shear stress and turbulent kinetic energy
k = turbulent kinetic energy
Re = Reynolds number
U = mean velocity
u',v',w' = components of turbulent fluctuation velocity
ρ = density
τ = turbulent shear stress

Subscripts

j = jet
o = outer stream

Other symbol

$\langle\ \rangle$ = time-average

Introduction

SINCE the development of the mixing length concept by Prandtl[1] in 1925, analytical investigations of turbulent flow phenomena generally have used some form of a locally dependent shear stress model. Today a number of such models are available, yet none can be applied to the analysis of a wide variety of turbulent flow problems with reasonable confidence. It is obvious that a more fundamental approach is needed. One such approach involves the use of the turbulent kinetic energy equation.

One of the first investigators to consider this approach was Nevzgljadov, cited in Ref. 2, who proposed to select the mean velocity, the mean pressure, and the turbulent kinetic energy as the independent variables in a turbulent boundary layer analysis. He further proposed a relation in which the

Received December 24, 1969; revision received February 2, 1970. The research reported in this paper was sponsored by Arnold Engineering Development Center, Air Force Systems Command, Arnold Air Force Station, Tenn., under Contract F40600-69-C-0001 with ARO, Inc. Further reproduction is authorized to satisfy the needs of the U.S. Government.

* Research Engineer, Research Branch, Rocket Test Facility. Associate Member AIAA.

† ARO Consultant and Assistant Professor, University of Missouri, Rolla, Mo. Member AIAA.